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The Construction of a Natural Language and Graphics Interface;
Results and Perspectives from the ACORD Project, Springer, 1992, p. 47-64.

Abstract

This article suggests that linguistic observations can be captured by an axiomatic system ; models satisfying the axioms can be compared with sentences specified by the grammars in order to control their descriptive adequacy. The authors show how this can be performed with a UCG type grammar.

See also

Karine Baschung, Gabriel G. Bès, Thierry Guillotin. “French Unification Categorial Grammars.” *The Construction of a Natural Language and Graphics Interface; Results and Perspectives from the ACORD Project*, Springer, p. 23-46, 1992.
https://hal.archives-ouvertes.fr/halshs-00371433

Gabriel G. Bès. « La phrase verbale noyau en français. » *Recherches sur le français parlé*, Publications de l’Université de Provence, 1999, n° 15, p. 237-358.
https://hal.archives-ouvertes.fr/hal-01005527

CHAPTER 4

The Control of UCG Grammars

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1 Introduction

The work on the control of grammars arises from our contribution to the ACORD project which was concerned with (in close cooperation with LdM) the adaptation to French of two models of grammars (GPSG and UCG).

The control of grammars consists of four parts : (i) a descriptive metalanguage expressed as a set of axioms Θ ; (ii) the linearisation of Θ in terms of the notion of a *constructor* ; (iii) a *transition relation* between some constructor associated with Θ and a grammar $\mathcal{G}$; it is this relation which allows the comparison of models of Θ with the sentences specified by $\mathcal{G}$. The set of axioms Θ of the descriptive metalanguage is the formalised knowledge about the object language. It captures observations about the object language but it differs in function and formalism from the particular grammar models which are intended to be associated with the same object language.

Control of grammars has both practical and theoretical goals. The practical goals are related to reliability, reusability and modularity requirements on linguistic observations.

Reliability is a necessary condition of any NLP practical application. But our claim is that so far nobody knows explicitly what the coverage of a parser or a generator is. Demos and actual specifications are, at best, hints indicating roughly what the system can or should be capable of. When a lot of money is spent in the implementation of grammars and parsers, it is a big disappointment to find in a haphazard way obvious but completely unforeseen counter-examples to some system. This arises not from the fact that an 'actual grammar cannot cover all the language' -a point which nobody denies- but from the fact that examples which 'work' say very little about the descriptive power of the grammar. Control of grammars can thus contribute to enhance the reliability of NL products and to monitor extensions of their coverage.

We do not know of any grammar of the UCG or LFG type of any language having been extended in a modular way by several different teams. Nor are we aware of any systematic tailoring of grammars of this type in order to re-use them for particular specific applications. Even if we do not discuss here the usefulness of the notion of grammar, and even if we do not conjecture that UCG or LFG grammars are definitively neither modular nor reusable, we do conjecture that the reusability and modularity goals can be satisfied more efficiently if purely axiomatic knowledge about linguistic data of the metalanguage type is carefully distinguished from grammar type knowledge, the latter being expressed by 'constructive rules' (see below).

On the theoretical level, we found in general strong scepticism about the notion of descriptive metalanguage. Two main criticisms are generally advanced :

(i) There are no 'facts' independent of a particular theory.
(ii) Knowledge about an object language can be expressed only by a grammar. A metalanguage is a grammar G_j that duplicates the grammar G_i which is intended to be verified.

With the assumption that (computational) linguistics is an empirical science, criticism (i) must be examined in terms of the states of data in empirical[1] sciences.

These conform to a general pattern in which hypothetical objects are distinguished from the data they must account for. Low level theorems (or predictions) deduced from deeper ones and from the more general hypotheses of the theory must be compared with observations or lower level generalisations of observations, which are formulated in some more or less explicit language. This crucial point is condensed in the following citation from (Bunge 69) [2].

"Given a set of theoretical predictions and a set of empirical data, it is necessary to decide whether the two sets match [...] Theories cannot be compared with observational data directly. Before being able to compare a set of theoretical predictions with an empirical report, we must make them comparable, formulating them in the same language [...] At first glance, there are two means of achieving this goal. Either we translate theoretical predictions into some observational language, or, inversely we translate empirical data into the language of the theory. In fact, the two transformations are effected at the same time"(p.537-538).

Seismograms are different from the predictions of some vulcanological theory. The two must be compared, and it is through this comparison that a particular vulcanological theory is confirmed or not.

We do not understand by what token we may (or can) not have in (computational) linguistics objects which have the same relation with respect to theoretical predictions of grammars as seismograms with respect to predictions of vulcanological theories.

Observe that we are not claiming that linguistic observations are naive constructs with no theoretical background, nor that they follow immediately from sensory experience, nor that they are monolithically invariant. We do not say either that it is necessary to collect *first* all the data in order to do *afterwards* all the theoretical work on grammatical models and on particular grammars.

In much more simple and practical terms we claim that there is a common pattern of problems that GB, LFG, GPSG, UCG, CUG, FUG, TUG, pure CG, CCG and many others recognize as such, and that there is a consensus of agreement about them, even if the statement of the conditions underlying each one may be quite difficult.

It may be that (computational) linguistics is unique with respect to other empirical sciences in the fact that there are always a great number of interlocking variables which underlie the descriptive statements of any set of observations. A different descriptive predicate must be associated with each variable, and the overall picture of interactions between all the intervening

[1]Empirical does not have here any ontological status. It simply says that (computational) linguistics must be ranged with physics, chemistry or biology, and not with mathematics or logics ; i.e., that in one way or another, (computational) linguistic hypotheses can be falsified (tested or verified) by observations (or facts or data).

[2]The citation is actually an English re-translation of the Spanish edition of the book (Bunge, Mario. *La investigacion cientifica.* Barcelona, Ariel, 1969)

variables is quickly lost. But this state of affairs must be interpreted as pointing to the necessity of a descriptive metalanguage rather than the opposite.

Criticism (ii) asserts the uselessness of a metalanguage ; this, at best, doubles the work that is necessary in any case.

The validity of criticism (ii) depends on the definition of *grammar.* Besides formal properties, we suggest that grammars, when referring to such objects as LFG, UCG, GPSG, FUG, HPSG particular grammars, are intended to satisfy at least three basic requirements :

(a) Constructive rules
(b) Integration with parsing and generation
(c) Characterisation of NL

By (a) we understand that the rules of the grammar are not only inferencing rules but also actual processes defining effective construction of structures from other structures.

By (b) we understand that grammars must be integrated in parsing and generation mechanisms in efficient ways.

By (c) we understand that grammars must contribute to the definition of NL. We expect from a grammar the expression of NL characteristics.

Say we label the grammars satisfying (a) to (c) *r-grammars (restricted grammars).* We can reword criticism (ii) and obtain the following.

(iii) Knowledge about an object language can be expressed only by r-grammars.

The assumption of our work on control of grammar is that (iii) is not true. Independently from labels, the metalanguage we proposed is not an r-grammar, because it does not satisfy any of the above (a) to (c) requirements.

Axioms are not constructive rules. That is, they say nothing about the concatenation of leaves, the specification of trees, etc. A set may be specified in a predicate notation (as in (a)) or by recursive rules (as in (b)) ; the (a) type corresponds to Θ ; the (b) type to $\mathcal{G}$.

(a) $\{x \mid x$ is a positive integer less than n$\}$
(b) (i) $1 \in E$
  (ii) If $x \in E$ and x is less than n-1, then $n + 1 \in E$
  (iii) nothing else is in E

Axioms express declarative knowledge which is not intended to be integrated in their actual form as the declarative knowledge effectively used in NL processes such as parsing or generation. The notion of procedural 'efficiency' is thus alien to them.

Axioms are not meant to say anything clever about NL. Rather, they are intended, ideally, to present in an orderly, explicit and modular manner, observations about NL. Because the metalanguage is a formalised object, it is entirely possible that careful descriptive axiomatisation of linguistic observations will allow the formulation of real and valid generalisations about NL, i.e, formal statements which can be empirically falsified with respect to formalised or formalisable observations and from which it will be formally possible to deduce low level descriptive axioms.

To sum up : a metalanguage has a different function from an r-grammar. Metalanguage axioms express a durable and reusable source of knowledge about language. They may 'provide' the actual declarative knowledge effectively used in language performance, but they must not be confused with the declarative knowledge which directly underlies language use.

2 The Metalanguage

The aim of the metalanguage is to give an axiomatic description of the sentence. Here we consider only the simple sentence ; in particular noun phrases are not analysed, that is they are treated as atomic.

We will consider two stages in this axiomatisation : in the first one, the lexicon, the formalised properties are merely properties of the phonological representations of individual items, not of strings. The second stage introduces properties of the sentence (as a string) ; for example properties which involve the order of the phonological representations.

2.1 The Lexicon

It consists of four components : (a), (b), (c), (d).

We start with two sets :

(a) E (the phonologies), and
(b) T (the valencies).

a, b, ...will denote elements of E ; t, t_1, ...elements of T.

We fix a set of unary *primitive lexical predicates* (subsets) of E :

(c) $\varphi_1, \ldots, \varphi_n$
$\sigma_{t1}, \ldots, \sigma_{tp}$ $(t_1, \ldots, t_p \in \mathrm{T})$

Boolean combinations of these primitive predicates will be called *lexical predicates*; they are denoted by φ, ψ, ...We write $\varphi(x)$ or $x \in \varphi$ if x is an element of the subset of E denoted by φ.

$\wedge$, $\vee$, $\longrightarrow$, $\longleftrightarrow$, $\sim$ are the classical Boolean connectors (conjunction, disjunction, implication, equivalence, negation), $\forall$ and $\exists$ are the classical quantifiers.

(d) A set of *lexical axioms* of one of the following forms.

disjointedness :: φ and ψ are disjuncts i.e. : $\forall a \in \mathrm{E} \sim (\varphi(a) \wedge \psi(a))$

inclusion :: $\forall a \in \mathrm{E}\ \varphi(a) \longrightarrow \psi(a)$

2.2 Example 1

E is a set of phonological representations of words or complex noun phrases.

T = {nom, obj, dat, mod, _}, consists of the valencies : *nominative, objective, dative, modifier* ; we add '_' which is useful to denote the absence of real linguistic affectation of valency.

Primitive lexical predicates

verb

- fin (finite verb)
- imp (imperative verb)
- part (participle)

aux (auxiliary)

cl (clitic)

- clnom (nominative clitic)
- clobj (objective clitic)
- cldat (dative clitic)

np (noun phrase)

- np (nom or obj)
- npdat

pp (prepositional phrase), ne, pas

lex (lexical element), wh (interrogative element), lexne (negative lexical element)

sub_t, for t $\in$ T and t $\neq$ _ (verb subcategorisation)

Lexical axioms

(1) φ and ψ are disjoint for every pair of distinct φ and ψ in {fin, imp, part, clnom, clobj, cldat, np (nom or obj), npdat, pp, ne, pas}.

(2) φ and ψ are disjoint for every pair of distinct φ and ψ in {lex, wh, lexne}.

(3) $\forall$x $\in$ E (verb(x) $\longleftrightarrow$ fin(x) $\vee$ imp(x) $\vee$ part(x))

(4) $\forall$x $\in$ E (cl(x) $\longleftrightarrow$ clnom(x) $\vee$ clobj(x) $\vee$ cldat(x))

(5) $\forall$x $\in$ E (np(x) $\longleftrightarrow$ np(nom or obj) $\vee$ npdat(x))

(6) $\forall$x $\in$ E ((np(x) $\vee$ pp(x)) $\longleftrightarrow$ (lex(x) $\vee$ wh(x) $\vee$ lexne(x)))

Obviously such an example can be expanded and refined very simply by extending the components (a), ..., (d).

2.3 The Sentence

We will consider now properties for strings of the form

$u = u_0, u_1 \ldots u_n$

where each u_i is a pair $<a_i, t_i>$ with $a_i \in E$ and $t_i \in T$. We say that a_i is the phonological representation of u_i and t_i its valency. x, x', y, ...will denote pairs of $<a, t>$.

A sentence will be a string u which satisfies a set of axioms, $\Theta(u)$. But we do not want, and, in fact we do not need, to write here an explicit and definitive set, $\Theta(u)$, of such axioms. Firstly, any such axiomatic system must not be considered as a 'definitive' one, but rather as a very flexible tool. Secondly, from a theoretical point of view, the form of the axioms we introduce in $\Theta(u)$ is much more important than what they contain. But we assume that the characterisation of the simple sentence of the French language does require axioms of more complex form than those of types (I) ...(VI) below.

To define the form of these axioms in Θ we adopt the following conventions. Let $u = u_0$, $u_1 \ldots u_n$ be a string of the previous form.

(1) If φ is a lexical predicate we denote $\varphi(i)$ if the phonology of u_i satisfies φ.

(2) For each valency t, we denote $t(i)$ if t is the valency of u_i.

(3) We distinguish a lexical predicate denoted by ζ (in actual applications ζ will be the verb) and we denote by

$F(i)$ the formula $\forall\ k\ (\zeta(k) \longrightarrow i < k)$, and

$B(i)$ the formula $\forall\ k\ (\zeta(k) \longrightarrow k < i)$.

Then $F(i)$ (resp. $B(i)$) is true in u iff the place i is before (resp. after) every place where ζ occurs.

(4) We denote by p, q, ...every Boolean combination of *lexical*, t, F and B predicates.

2.4 Axiom Types for Sentences

(I) Order Type

$[p < q](u) :: \forall i, j\ (p(i) \wedge q(j) \longrightarrow i < j)$
([p < q] is true in u if for any positions i and j in u, if p is satisfied at i and q at j, then i is before j)

(II) Unicity type :

$\ll p \gg(u) :: \forall i, j\ (p(i) \wedge p(j) \longrightarrow i = j)$
(There exists at most one place in u where p is satisfied)

(III) Incompatibility type

$[p \text{ X } q]\ (u) :: \forall i, j \sim (p(i) \wedge q(j))$
(p and q cannot be both satisfied in u)

(IV) Pure implicational type

$[p \Longrightarrow q](u) :: \forall i\ (p(i) \longrightarrow q(i))$
(Every position in u, satisfying p, satisfies q)

(V) Absolute existential type

$[\exists p](u) :: \exists i\ p(i)$
(There exists some place in u satisfying p)

(VI) Relative existential type

$[\exists p \Longrightarrow \exists q](u) :: \exists i\ p(i) \longrightarrow \exists j\ q(j)$
(If p is satisfied at some place in u, then q is also satisfied at some place in u).

Remarks

(1) For any p, q we have, in every u

$$[p < q] \wedge [q < p]) \longleftrightarrow p \ X \ q$$

(2) The relation $p < q$ is not transitive but we have for every p, q, r, in every u

$$(p < q \wedge q < r \wedge \exists q) \longrightarrow p < r$$

2.5 Example 2

We describe here a small axiomatic system $\Theta(u)$. Its lexicon is a reduction of the one in our Example 1. To simplify the notation the Boolean conjunction symbol, $\wedge$, will often be omitted. For example, the expression *objwhF* will denote the predicate $obj(i) \wedge wh(i) \wedge F(i)$. $\Theta(u)$ will consist of the following primitive lexical predicates and axioms.

Primitive lexical predicates

ζ (= verb), np, pp, cln, wh, lex

Lexical axioms

(1) $\forall x \sim(\varphi(x) \wedge \psi(x))$, for any distinct φ and ψ among ζ, *np*, *pp* and *cln*.
(2) $\forall x \sim (lex\ (x) \wedge wh(x))$
(3) $\forall x\ np(x) \longleftrightarrow (lex(x) \vee wh(x))$

Pure implicational axioms

(4) $\sim\zeta \Longrightarrow np \vee pp \vee cln$
(5) $np \Longleftrightarrow nom \vee obj$
(6) $cln \Longrightarrow B$
(7) $objF \Longrightarrow wh$
(8) $nomB \Longrightarrow lex$

Unicity axioms

(9) $\ll \zeta \gg$
(10) $\ll cln \gg$
(11) $\ll nom \gg$

(12) $\ll$ obj $\gg$

Incompatibility axioms

(13) nomB X cln
(14) objwhB X cln
(15) nomwhF X cln

Order axioms

(16) nomB $<$ objB
(17) cln $<$ objB
(18) objF $<$ nomF
(19) ppF $<$ (nom $\vee$ obj)
(20) (nom $\vee$ obj $\vee$ cln) $<$ ppB

Absolute existential axioms

(21) $\exists\zeta$
(22) $\exists$(nom $\vee$ cl)

Relative existential axiom

(23) $\exists$nomB $\Longrightarrow$ $\exists$objwhF

The models of Θ. It can be seen that $\Theta(u)$ is true iff u is a string of one of the following types

(1)	whobj	verb	nomlex	
(2)	npnom	verb		
(3)	npnom	verb	npobj	
(4)	whobj	npnom	verb	
(5)	lexnom	verb	clnom	
(6)	lexnom	verb	clnom	lexobj
(7)	whobj	lexnom	verb	clnom
(8)	verb	clnom		
(9)	verb	clnom	lexobj	
(10)	whobj	verb	clnom	

plus a potentially infinite family $\mathcal{M}$:

$$< x_1, \ldots, x_i, \mathrm{S}, y_1, \ldots, y_j >$$

where S is a model of type (1) to (10) and the x's and y's are *pp*.

More formaly a model of type (1), for example, is a string

$u = (u_0, u_1, u_2) = (<a_0, t_0>, <a_1, t_1>, <a_2, t_2>)$

such that a_0 is a *npwh*, t_0 = *obj*, a_1 is a *verb*, $t_1 = \_$, a_2 is *nplex* and t_2 = *nom*.

The proof that the set of all the models of the axiom system Θ is exactly given by the above list plus $\mathcal{M}$ is rather intricate. It is done by disjunction in various instances of the kernel sentences i.e. sentences without *pp*, plus induction on the length of the sentences for recursive sentences (sentences with an arbitrary number of *pp*). The method of transition relations that we will introduce in the section 4 will avoid detailing a complete list of the models of an axiomatic system Θ, when this system is compared with a grammar $\mathcal{G}$.

3 Constructibility

The axiomatic systems give static descriptions of sentences in a way completely independent of the various dynamic systems (like grammars) that may be used to *construct* these sentences.

Def 1. Let $\mathcal{G}$ be a grammar and Θ an axiomatic system. We write $\mathcal{G}(u)$ whenever u is a string which can be constructed as a (complete or terminal) sentence by $\mathcal{G}$, and we say that $\mathcal{G}$ and Θ are equivalent iff

$$\forall u\ (\mathcal{G}(u) \longleftrightarrow \Theta(u))$$

The problem which arises is, given $\mathcal{G}$ and Θ, how to prove that $\mathcal{G}$ and Θ are equivalent.

3.1 The Central Kernel Property

As $\mathcal{G}$ is a dynamic system, to work on a terminal sentence u of $\mathcal{G}$ requires to know all the history of the contruction process for u. Thus it is not at all sufficient to take into account only the terminal sentences of $\mathcal{G}$ but we have to incorporate in our theory all the substrings which are constructed by $\mathcal{G}$ at any intermediate stage.

Def 2. We will write $*\mathcal{G}(u)$ if u is a string constructed by $\mathcal{G}$ at some stage (terminal or not).

Notation. Every string u of length > 1 will be represented by

$$u = x'w = vx$$

where x' is the first element and x is the last one of u.

The following property reflects the fact that u 'has a history', or in other words that it is recursively constructed.

The central kernel property. A predicate $P(u)$ possesses the central kernel property, CKP, iff :

$$\forall(u)\ (P(u) \longrightarrow P(v) \vee P(w)).$$

Clearly if $\mathcal{G}$ is of the type UCG for example, $*\mathcal{G}(u)$ has the CKP. This is due to the fact that in this type of grammar, the phonological representations are constructed by concatenation of adjacent elements going up from the verbal entry (recall that we have not analysed here the noun phrases). (For example in the contruction of *[John gives a book]*, at least one of the substrings *[John gives]* or *[gives a book]* is produced at an intermediate stage, but not the substring *[John a book]*.) In general, an axiomatic system does not have the CKP, because of the existential axioms (pure or relative) it may contain.

Notation. Θ' and Θ"will denote two parts of Θ such that

(a) $\forall u\ (\Theta(u) \longleftrightarrow \Theta'(u) \vee \Theta''(u))$, and

(b) $\Theta'(u)$ has the CKP

This decomposition of Θ is not unique . For example, we may choose obviously Θ' as the universal part of Θ, and Θ" as its existential part. But, under some assumptions it is possible to leave also some existential types axioms in Θ'.

Proposition 1. If Θ' contains two absolute existential axioms, $\exists p$ and $\exists q$, and if p and q are disjoint, then Θ' does not have the CKP (disjointedness means that $\forall a \sim (p(a) \vee q(a))$ as in 2.1)

Proof. If Θ' has the CKP then Θ' would have a model u, of length >1, such that, for example, x satisfies p, x' satisfies q, and any other elements of u does not satisfy p nor q. Then we would have $\sim \Theta'(v)$ and $\sim \Theta'(w)$; and this is impossible by CKP.

Proposition 2. Suppose that $\exists\zeta$ and $\exists p \Longrightarrow \exists q$ are axioms of Θ' (p and q, disjoints). If Θ' has a model u in which it is true that

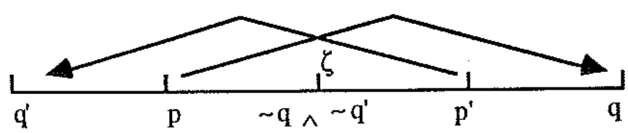
$$\exists ij\ (p(i) \wedge B(i) \wedge q(j) \wedge B(j) \vee \forall k\ (k < i \longrightarrow \sim q(k)))$$

then Θ does not have the CKP.

We symbolise this situation (and its dual situation, for F) by the following figure :

Proof. If Θ' has the CKP, then Θ' would have a model u such that x' satisfies ζ, x satisfies q, p is satisfied in v and q is not satisfied in v. Then v is not a model Θ' (because $\exists p \Rightarrow \exists q$ is false in v) and w is not a model of Θ' (because $\exists\zeta$ is false in w) ; but this contradicts the CKP hypothesis.

Proposition 3. Suppose that $\exists\zeta$, $\ll \zeta \gg$, $\exists p \Longrightarrow \exists q$ and $\exists p' \Longrightarrow \exists q'$ are in Θ'. (p, q disjoints and p', q' disjoints). If Θ' has a model symbolised by



then Θ' has not the CKP.

The proof is very similar to the proof of the previous proposition.

Proposition 4. Suppose that Θ' satisfies the following conditions.

(i) $\exists\zeta$ and $\ll \zeta \gg$ are in Θ'.

(ii) For every $p \neq \zeta$, $\exists p$ is not in Θ'.

(iii) For every $\exists p \Longrightarrow \exists q$ in Θ', the formulas $qB < pB$ and $pF < qF$ are logical consequences of Θ'.

(iv) For every pair $\exists p \Longrightarrow \exists q$ and $\exists p' \Longrightarrow \exists q'$ in Θ', the formulas $qB < p'B$ and $pF < q'F$ are logical consequences of Θ'.

Then Θ' has the CKP.

Proof. Suppose that Θ' does not have the CKP, then Θ' has a model u such that neither v nor w is a model of Θ'. Thus there exist two (necessary) existential axioms A and A' such that A is not true in v and A' is not true in w.

By (i) and (ii) we have only three cases :

1st case. A' is $\exists\zeta$ and A is of type $\exists p \Longrightarrow \exists q$ and then u is a model of type I, but such models do not exist by condition (iii).

2nd case. Similar to the 1st case, with type A and A' exchanged.

3rd case. A is of type $\exists p \Longrightarrow \exists q$ and A' is of type $\exists p' \Longrightarrow \exists q'$. Then u is a model of type (III) but such models do not exist by condition (iv).

3.2 Constructors

To go on in the modelization of the recursive construction of sentences in a UCG type grammar, we introduce now the concept of constructor.

Def. A pair of predicates $\mathcal{F}(x',w)$, $\mathcal{B}(v, x)$ is said to be a *constructor* for Θ if it satisfies the following condition.

(CONST) If the lenght of u > 1, then $\Theta'(u) \longleftrightarrow ((\Theta'(w) \wedge \mathcal{F}(x', w) \vee (\Theta'(v) \wedge \mathcal{B}(v, x))$

Intuitively $\mathcal{F}$ (resp. $\mathcal{B}$) will express under what conditions it is possible to add a new element in front (resp. at the end) of a model of Θ' to obtain a new model of Θ'. The condition CONST implies that every model not reduced to the verb is constructed from a model by adding an element to it on the left by $\mathcal{F}$ or on the right by $\mathcal{B}$. Thus CONST implies obviously that Θ' has the CKP.

Proposition 5. Θ' admits a constructor if Θ' has the CKP.

Proof. If Θ' has the CKP, then the pair $<\mathcal{F},\mathcal{B}>$ defined by

$\mathcal{F}(x', w) \longleftrightarrow \Theta'(u) \longleftrightarrow \mathcal{B}(v, x)$

is a constructor of Θ'.

3.3 Translation from Θ' to $< \mathcal{F}, \mathcal{B} >$

We suppose here that Θ' has the CKP and that the axiom $\exists\zeta$ is in Θ. Then by Proposition 1, Θ' does not contain any other absolute existential type axiom and by Proposition 5, Θ' admits a constructor. We will see that the constructor defined in the proof of the Proposition 5 can be derived from the axioms of Θ'in a uniform way.

Let us denote by $\perp$ (resp. $\top$) any logically false (resp. true) formula. For each Boolean combination of lexical, *t-s* , *F* and *B* predicates, *p*, and for each axiom, *A*, of Θ' (except $\exists\zeta$) we defined left translation $^*p$ and ^0A and right translation p^* and A^0 by the following rules.

(R_0) $^*p$ is p in which each F is changed into $\top$ and each B is changed into $\perp$.

p^* is p in which each F is changed into $\perp$ and each B is changed into $\top$.

(A^0, ^0A) are related to some $<\mathcal{F},\mathcal{B}>$ admitted by Θ'(see Proposition 6).

(R_1) If A is p < q, then

$A^0(v, x) :: p^*(x) \longrightarrow \sim \exists\, j \in v\ q(j)$

$^0A(x, w) :: {}^*q(x) \longrightarrow \sim \exists\, j \in v\ p(j)$

(R_2) If A is $\ll$ p $\gg$, then

$A^0(v, x) :: p^*(x) \longrightarrow \sim \exists\, y \in v\ p(y)$

$^0A'(x, w) :: {}^*p(x) \longrightarrow \sim \exists\, y \in w\ p(y)$

(R_3) If A is p X q, then

$A^0(v, x) :: [p < q]^0 \wedge [q < p]^0$

$^0A(x, w) :: {}^0[p < q] \wedge {}^0[q < p]$

(R_4) If A is p $\Longrightarrow$ q, then

$A^0(v, x) :: p^*(x) \longrightarrow q^*(x)$

$^0A(x,w) :: {}^*p(x) \longrightarrow {}^*q(x)$

(R_5) If A is $\exists$p $\Longrightarrow$ $\exists$q, then

$A^0(v, x) :: p^*(x) \longrightarrow \exists y \in v\ q(y)$

$^0A(x, w) :: {}^*p(x) \longrightarrow \exists y \in w\ q(y)$

Proposition 6. Let $\mathcal{F}(x, w)$ (resp. $\mathcal{B}(v, x)$) be the conjunction of all the left (resp. right) translations of the axioms of Θ' (except $\exists\zeta$).

If Θ' has the CKP then $<\mathcal{F},\mathcal{B}>$ is a constructor of Θ'.

Proof. It can be verified that for each string u and each axiom A :

(a) $\quad A(u) \longrightarrow {}^0A(x, w) \wedge A^0(v, x)$

(b) $\quad A(w) \wedge {}^0A(x, w) \longrightarrow A(u)$, and

(c) $\quad A(v) \wedge A^0(v, x) \longrightarrow A(u)$

Then from (a), (b), (c) and the CKP we deduce that $<\mathcal{F},\mathcal{B}>$ is a constructor of Θ'.

3.4 Example 3

We come back to the system Θ introduced in the Example 2. Clearly Θ has not the CKP because it contains two absolute existential axioms ($\exists\zeta$ and $\exists(cl \vee nom)$) (cf. Proposition 1). Let Θ'_1 be Θ minus the axiom $\exists(\mathrm{cl} \vee \mathrm{nom})$ and Θ'_2 be Θ'_1 minus the axiom $\exists nomB \Longrightarrow \exists obj\ whF$. By Proposition 4, Θ'_1 and Θ'_2 have the CKP.

If, for example, A is the axiom $cln \Longrightarrow B$, it is easy to verify from (R_0) and (R_4) that

$A^0(v, x) \longleftrightarrow \top$, and

${}^0A(x, w) \longleftrightarrow \sim \mathrm{cln}\ (x)$

Similary, if A is $nomB < objB$, then from (R_0) and (R_1) we have

$A^0(v, x) \longleftrightarrow (\mathrm{nom}(x) \longrightarrow \sim \exists y \in v\ (\mathrm{obj}(y) \wedge B(y)))$, and

${}^0A(x, w) \longleftrightarrow \top$

If A is $\exists nomB \Longrightarrow \exists objwhF$, then, from (R_0) and (R_5) we have

$A^0(v,x) \longleftrightarrow (\mathrm{nom}(x) \longrightarrow \exists y \in v\ (\mathrm{obj}(y) \wedge \mathrm{wh}(y) \wedge F(y)))$, and

${}^0A(x,w) \longleftrightarrow \top$

It is practically convenient to summarize all the information contained in $\mathcal{F}$ and $\mathcal{B}$ by the following tables (tables for Θ'_1 and Θ'_2 will differ only by the 2nd line of their $\mathcal{B}$-tables).

$\mathcal{B}$ - table for Θ'_2 (and Θ'_1)

B	-	+
cln	cln, nom, objB, ppB	
npnomlex	cln, nom, objB, ppB	objwhF (For $\theta 1$ only)
npobjwh	cln, obj, ppB	
npobjlex	obj, ppB	
pp		

$\mathcal{F}$ - table for Θ'$_2$ (and Θ'$_1$)

F	-	+
npnomlex	nom, obj F, ppF	
npnomwh	nom, obj F, cln, ppF	
npobjwh	obj, ppF	
pp		

Conventions for table reading

(1$\mathcal{B}$) $\mathcal{B}(v,x)$ implies that x satisfies one of the predicates of the left column in the $\mathcal{B}$-table.

(2$\mathcal{B}$) If the following conditions are satisfied

(a) p is a predicate in the left column of the $\mathcal{B}$ - table,
(b) $q_1, \ldots, q_k$ are all the predicates in the line of p and the column -
(c) $r_1, \ldots, r_l$ are all the predicates in the line of p and the column +, and
(d) x satisfies p,

then, $\mathcal{B}$(v, x) is true iff

(i) For each p_i and each y in v $q_i(y)$ is false and
(ii) For each r_i there exists some y_i in v such that $r_i(y_i)$ is true.

For (1$\mathcal{F}$) and (2$\mathcal{F}$) the same conventions as (1$\mathcal{B}$) and (2$\mathcal{B}$) are valid, with $\mathcal{B}$ changed in $\mathcal{F}$, and v in w.

4 Transition between Metalanguages and Grammars

We consider here a grammar $\mathcal{G}$ of UCG type and we introduce the following notations and conventions.

(1) S, S'...will denote signs of $\mathcal{G}$, and, for each atomic phonological representation a taken into consideration, S_a will denote a sign of phonology a, in $\mathcal{G}$.

(2) The rules of $\mathcal{G}$ are arranged into two classes. $\mathcal{B}\_t$ rules (i.e. Backward application rules) which consume the valency t and concatenate to the right of the verbal sign, and, similarly, $\mathcal{F}\_t$ rules (Forward application rules). $\mathcal{B}\_t$ rules are denoted by S $+_t$ S_a and $\mathcal{F}\_t$ rules by S_a $+_t$ S

(3) To each verbal sign S is associated a string, $u = ph(S)$, of pairs $x = <a, t>$ (atomic phonological representation + valency, if it is relevant) such that

(i) If S' = S $+_t$ S_a
v = ph(S) and x = $<a, t>$
then ph (S') = vx

(ii) if S' = $S_a +_t$ S
w = ph(S) and x' = $<a, t>$,
then ph(S') = x'w

(4) We denote by *Term* the set of all the terminal signs of $\mathcal{G}$ (e.g. signs in which all the valencies are consumed).

(5) Finally we have by definition, for all u

(i) $*\mathcal{G}$(u) iff there exists a sign S such that ph(S) = u, and

(ii) $\mathcal{G}$(u) iff there exists a sign S in Term(S) such that ph(S) = u

4.1 Transition relations

We recall that we suppose that $\Theta(u) \longleftrightarrow \Theta'(u) \wedge \Theta''(u)$ where $\Theta'(u)$ has the CKP, the axiom $\exists\zeta$ is in $\Theta'(u)$, and $<\mathcal{F},\mathcal{B}>$is a constructor of Θ'.

To obtain a criterion of equivalence between Θ and $\mathcal{G}$ we introduce *the notion of transition relation.*

Def. A *transition relation* (between $<\mathcal{F},\mathcal{B}>$ and $\mathcal{G}$) is a predicate $\mathcal{R}(u, S)$, which satisfies the following conditions for every string $u = x'w = vx$ and signs S and S_a.

(RT1) If a is a verb, then $\mathcal{R}$ ($<a, \_>$, S_a)

(RT2B) ($\mathcal{R}$(v,S) $\wedge$ S' = S $+_t$ S_a $\wedge$ x = $<a, t>$) $\longrightarrow$ $\mathcal{R}$(u, S')

(RT2F) $\mathcal{R}$(w,S) $\wedge$ S' = S_a $+_t$ S $\wedge$ x' = $<a$, t$>$) $\longrightarrow$ $\mathcal{R}$(u, S')

(RT3B) $\mathcal{R}$(v,S) $\longrightarrow$ ($\mathcal{B}$(v, $<a$, t$>$) $\longleftrightarrow$ S $+_t$ S_a is defined)

(RT3F) $\mathcal{R}$(w,S) $\longrightarrow$ ($\mathcal{F}$($<a$, t$>$, w) $\longleftrightarrow$ S_a $+_t$ S' is defined)

Proposition 7. If $\mathcal{R}(u, S)$ is a transition relation between $\mathcal{G}$ and $<\mathcal{F},\mathcal{B}>$, then

$\forall$u (Θ'(u) $\longleftrightarrow$ $*\mathcal{G}$(u))

If moreover, $\mathcal{R}$(u,S) satisfies

(RT4) $\mathcal{R}$(u,S) $\longrightarrow$ (Θ"(u) $\longleftrightarrow$ Term (S))

then Θ and $\mathcal{G}$ are equivalent.

Proof. The proof is straightforward by induction on the length of u using the statement that, for every u and S, $ph(S) = u$ implies $\mathcal{R}(u, S)$.

4.2 Example 4

We describe now another small axiomatic system Θ (specially oriented toward some problems of French negation)[3] and we will show a UCG grammar $\mathcal{G}$ and a transition relation $\mathcal{R}$ between this grammar and a constructor $< \mathcal{F}, \mathcal{B} >$ of Θ. Thus by the previous theorem, it will turn out that Θ and $\mathcal{G}$ are equivalent.

Primitive lexical predicates

ζ (= verb), np, ne, pas, lex, neg

Lexical axioms

(1) $\forall x \sim (\varphi(x) \wedge \psi(x))$ for any distinct φ and ψ among ζ, *np, ne, pas*

(2) $\forall x \sim (\mathrm{lex}\ (x) \wedge \mathrm{neg}(x))$,

(3) $\forall x\ (\mathrm{np}(x) \longrightarrow (\mathrm{lex}(x) \vee \mathrm{neg}(x)))$

Pure implicational axioms

(4) $\sim \zeta \Longrightarrow \mathrm{np} \vee \mathrm{ne} \vee \mathrm{pas}$

(5) $\mathrm{np} \Longleftrightarrow \mathrm{nom} \vee \mathrm{obj}$

(6) $\mathrm{ne} \Longrightarrow \mathrm{F}$

(7) $\mathrm{pas} \Longrightarrow \mathrm{B}$

(8) $\mathrm{nom} \Longrightarrow \mathrm{F}$

(9) $\mathrm{obj} \Longrightarrow \mathrm{B}$

Unicity axioms

(10) $\ll \zeta \gg$

(11) $\ll \mathrm{ne} \gg$

(12) $\ll \mathrm{pas} \gg$

(13) $\ll \mathrm{obj} \gg$

(14) $\ll \mathrm{nom} \gg$

Incompatibility axiom

(15) pas X neg

Order axioms

(16) nom $<$ ne

(17) pas $<$ obj

Relative existential axioms

(18) $\exists \mathrm{pas} \Longrightarrow \exists \mathrm{ne}$

(19) $\exists \mathrm{neg} \Longrightarrow \exists \mathrm{ne}$

[3]On this and related points, see G.G. Bès and K. Baschung "Filtres dans une grammaire catégorielle". To appear in A. Lecomte (ed.) *Proceedings of the Workshop on Word Order in Categorial Grammar*, Clermont-Ferrand, May 25-27, 1990.

(20) $\exists$ne $\Longrightarrow$ $\exists$(pas $\vee$ neg)

Absolute existential axioms

(21) $\exists\zeta$
(22) $\exists$nom
(23) $\exists$obj

We define Θ' as the set of the axioms (1) to (19) plus (21) ; Θ" consists of (20), (22) and (23), and Θ = Θ' plus Θ". Clearly, from the Proposition 4 in 3.1, Θ'*(u)* has the CKP.

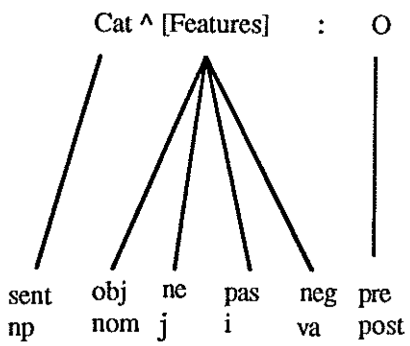
A contructor, $< \mathcal{F}, \mathcal{B} >$, of Θ'*(u)* is given by the following tables

B	-	+
pas	obj, pas	ne
objlex	nom, obj	
objneg	nom, pas, obj	ne

F	-	+
ne	nom, obj, pas, ne	
nomlex	nom	obj
nomneg	nom, pas	obj, ne

The grammar $\mathcal{G}$

Sign



Rules. Backward and Forward functional application.

Notation. Sem (*Semantics*) and *Ph* (*Phonology*) are omitted. We introduce the following conventions for denoting *Se signs* (i.e. signs whose *cat* value is *sent* and *lexical entries*).

Se signs

V : sent^[_, j, _, Va] / np^[nom, j, i, Va] : pre / np^[obj, j, i, Va] : post

V' : sent^[_, ne, _, Va] / np^[nom, ne, i, Va] : pre / np^[obj, ne, i, Va] : post

V" : sent^[_, ne, pas _] / np^[nom, ne, pas, _] : pre / np^[obj, ne, pas, _] : post

Lexical entries :

verbs : V

ne : V' / V : pre

pas : V" / V' : post

nplex : C / (C / np^[nom or obj, _, _, _] : _)

npneg : C / (C / np^[nom or obj, ne, i, neg] : _)

A transition relation between $< \mathcal{F}, \mathcal{B} >$ **and** $\mathcal{G}$. To describe more easily a transition relation $\mathcal{R}(u, S)$ between the previous $< \mathcal{F}, \mathcal{B} >$ and $\mathcal{G}$, we introduce the following notations. Given a verbal sign *Se*, *obj(Se)* [resp. *nom(Se)*] is *nc* (*not consumed*) if the valency *obj* [resp. *nom*] still occurs in the valency list of *Se*, and it is *c* (*consumed*) if this valency is already consumed. By *ne(Se)*, *pas(Se)* and *neg (Se)*, we denote the values in *Se* of the feature slots 2, 3 and 4 respectively.

Let $\mathcal{R}(u,\ Se)$ be the conjunction of the following (1) to (6).

(1) ne *is in* u $\longleftrightarrow$ ne(Se) = ne

(2) pas *is in* u $\longleftrightarrow$ pas(Se) = pas

(3) $\exists$x $\in$ u neg(x) $\longleftrightarrow$ neg(Se) = neg

(4) $\exists$x $\in$ u obj(x) $\longleftrightarrow$ obj(Se) = c and nom(Se) = nc

(5) $\exists$x $\in$ u nom(x) $\longleftrightarrow$ nom(Se) = obj(Se) = c

(6) *u* is reduced to a verbal entry $\longleftrightarrow$ *Se* is of type V

Then by the Proposition 7, $\Theta'(u) \longleftrightarrow *\mathcal{G}(u)$.

The proof that $\mathcal{R}(u,\ Se)$ is actually a transition relation can be made by verifying the *RT* of **Def** in 4.1 successively for each type of argument entry.

Finaly let us suppose that *Term* ($\mathcal{G}$) consists (by definition of $\mathcal{G}$) of all the signs such that

(1) nom(Se) = obj(Se) = c, *and*

(2) ne(Se) = j, *or*
ne(Se) = ne *and* pas(Se) = pas, *or*
ne(Se) = ne *and* neg(Se) = neg

The condition (*RT4*) of Proposition 7 in 4.1 is also verified. And thus Θ and $\mathcal{G}$ are proved to be equivalent.